STATIC AND FREE VIBRATION ANALYSIS OF LAMINATED COMPOSITE SKEW PLATE WITH AND WITHOUT CUTOUT

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF TECHNOLOGY

IN

MECHANICAL ENGINEERING

(MACHINE DESIGN AND ANALYSIS)

By

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ROURKELA - 769008, ODISHA, INDIA

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UNDER THE GUIDANCE OF

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CERTIFICATE

*This is to certify that the thesis entitled, "**STATIC AND FREE VIBRATION ANALYSIS OF LAMINATED COMPOSITE SKEW PLATE WITH AND WITHOUT CUTOUT**", submitted by **Mr. Girish Kumar Sahu** in partial fulfillment of the requirements for the award of degree of Master of Technology in Mechanical Engineering with specialization in **Machine Design and Analysis** at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance. To the best of my knowledge, the matter embodied in this thesis is original and has not been submitted to any other university/institute for the award of any degree or diploma.*

Date: *03rd Jun, 2013*

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Girish Kumar Sahu

ABSTRACT

Most of the structures generally under severe static and dynamic loading and different constrained conditions during their service life. This may lead to bending, buckling and vibration of the structure. Therefore, it is necessary to predict the static and vibration responses of laminated composite plates/skew plates precisely with less computational cost and good accuracy of these complex structures and. A suitable finite element model is proposed and developed based on first order shear deformation theory using ANSYS parametric design language (APDL) code. This is well that, the theory accounts for the linear variation of shear stresses along the longitudinal and thickness direction of the laminates. The model has been discretised using an appropriate four noded isoparametric element (SHELL181) from the ANSYS element library. The free vibration and bending responses are computed using Block-Lanczos and Gauss elimination algorithm steps. The responses like, transverse deflections, normal and shear stresses and natural frequencies of composite laminates are obtained through batch method of APDL code. The convergence test has been done of the developed model for all different cases and compared with those available published literature. Parametric effects (modular ratio, support conditions, ply orientations, number of layers, thickness ratio, geometry of cutout, cutout side to plate side ratio and skew angle) on the static and free vibration responses are discussed in detail.

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1. INTRODUCTION

1.1 Overview

A composite is a structural material that consists of two or more constituents that are combined at a macroscopic level which are insoluble in each other and differ in form or chemical compositions. There are two phases of composite exists namely, reinforcing phase and matrix phase. The reinforcing phase material may be in the form of fibers, particles or flakes and the matrix phase materials are generally continuous such as polymer, metal, ceramic and carbon [1]. The properties of the composite material largely depend on the properties of the constituents, geometry and distribution of the phases. The distribution of the reinforcement determines the homogeneity or uniformity of the material system. The strength and stiffness of fibre reinforced composite materials increases in the fibre direction due to the continuity nature of fibre [2]. Some more natural composites are available in the nature say wood, where the lignin matrix is reinforced with cellulose fibers and bones in which the bone-salt plates made of calcium and phosphate ions reinforce soft collagen. Composite systems include concrete reinforced with steel and epoxy reinforced with graphite/carbon/boron fibers etc. Similarly, the properties and behavior of composite materials are discussed by many authors [3] and [4].

1.2 Importance of present study

Composite laminates are formed by stacking layers of different composite materials and/or fibre orientation. By construction requirements, composite laminates have their planar dimension one to two orders of magnitude larger than their thickness. Therefore, composite laminates are treated as plate elements [5]. Laminated composite plates are being increasingly used in many engineering applications such as aerospace, marine, automobile, sports, biomedical, heavy machinery, agricultural equipment and health instrument as well as in the other field of modern technology due to their high strength to weight ratio, high stiffness to weight ratio, low weight, high modulus, low specific density, long fatigue life, resistance to electrochemical corrosion, good electrical and thermal conductivity and other superior material properties.

In many industries as discussed in the aforementioned point stress singular plates are used depending on the application, these are generally named as, cutouts. Cutouts are necessary for assembling the components, damage inspection, access ports, electrical lines and fuel lines, opening in a structure to serve as doors and windows, provide ventilation, to reduce weight and for accessibility to other parts of the structure. It is needed at the bottom plate for passage of liquid in liquid retaining structures. It is well known that these structures are exposed to the undesirable vibration, extra amount of deflection and many more during their service life and again these plate structures having cutout may change the responses considerably. As discussed earlier, the plates having the cutouts reduce the total weight which in turn affect the vibration response similarly it also reduces the total stiffness and the bending behavior changes automatically.

Many industries uses laminated plate are not square or rectangle, some of the cases skew plates is also used. In skew plates, the angle between the adjacent sides is not equal to 90˚. It may refer to oblique, swept or parallelogram. If the opposite sides of the skew plates are parallel, it is parallelogram and when their lengths are equal, it is called a rhombic plate. It is widely used in various mechanical, civil and aero structures. These structures can be found easily in modern construction in the form of reinforced slabs and stiffened plates. Such structures are commonly used as floors in bridges, ship hulls, buildings and in the construction of wings, tails and fins of aircrafts and missiles. However, the composite skew laminates may severe static and dynamic loading during their service life. Hence, for the designer’s quest to model these complex structural problems precisely and study the effect of cutout on the static and dynamic behavior of composite skew laminates with less computational effort.

In order to reduce the experimental cost and have clear idea on real life problems modeling and simulation type of work has got well importance. Some numerical methods invented time to time for the improvement of numerical approaches, like finite difference method (FDM), Ritz method and finite element method (FEM) [6]. From last few decades, FEM has got huge appreciation due to its applicability as well as preciseness. FEM proves to be a more versatile technique than all the other exiting method.

Based on the necessity many theories were developed in past to design and predict the responses of laminated composite plates [3].

(1) Equivalent single layer theory (2-D)
  (a) Classical laminated plate theory
  (b) Shear deformation laminated plate theory
(2) Three-dimensional elasticity theory (3-D)
  (a) Traditional 3-D elasticity formulation
  (b) Layerwise theory

The equivalent single layer (ESL) plate theory is derived from the 3-D elasticity theory by making suitable assumptions concerning the variation of displacements and stresses through the thickness of the laminate. These assumptions allow the reduction of a 3-D problem to a 2-D problem. In the three-dimensional elasticity theory, each layer is modeled as a 3-D solid. The simplest ESL plate theory is the classical laminated plate theory (CLPT), which is an extension of Kirchhoff (classical) plate theory to laminated composite plates. The next theory of ESL plate theory is the shear deformation laminated plate theory. The first order shear deformation theory (FSDT) extends the kinematics of CLPT by including a transverse shear deformation in its kinematics assumptions i.e., the transverse shear strain is assumed to be constant with respect to thickness coordinate. The higher order shear deformation theory (HSDT) provides a slight increase in accuracy relative to FSDT solution, at the expense of an increase in computational effort.

1.3 Objective

The objective of the present study is to develop a finite element (FE) model of laminated composite skew plates with and without cutouts (circular, rectangular and square) and analyze the static and free vibration behavior. The composite laminate was modeled in ANSYS finite element package and solved using ANSYS parametric design language (APDL) code. Effects of different elastic, geometric parameters, material properties, cutout geometries and skew angles on the static and vibration responses are obtained by using the developed FE model.

2. REVIEW OF LITERATURE

2.1 Historical development

The concept of composite material is very old. The use of reinforcing mud walls in houses with bamboo shoots glued laminated wood by Egyptians (1500 B.C.) and laminated metals in forging swords (A.D. 1800). In the 20th century, modern composites were used in the 1930s when glass fibers reinforced resins. Boats and aircraft were built out of these glass composites [1]. The pace of composite development was accelerated during World War II. Not only even more aircraft being developed and therefore, composites were more widely used in tooling, but the use of composites for structural and semi-structural parts. Since the 1970s, application of composites has widely increased due to development of new fibers such as carbon, boron and aramids and new composite systems with matrices made of metals and ceramics. Concrete is also a composite material and used more than any other man-made materials in the world. About 7.5 billion cubic meters of concrete are made each year. So, there is a need to study the static and vibration behavior of laminated composite structure precisely.

In recent years, many adequate theories/formulations have been proposed by the researchers to overcome the lacuna of the composite structures and advanced structural materials over conventional materials. The static and vibration responses of laminated composite plates have been investigated extensively by a number of researchers to fill the gap. However, the analysis of laminated skew composites considering cutout reported in the open literature is less in number. Some of the selected research findings are discussed in the following lines. For the sake of clarity, past work has been subdivided into static and vibration of laminated composite structures.

2.2 Static analysis of laminated composite structures

A C^0 continuous displacement finite element formulation of laminated composite plates under transverse loads is presented by Pandya and Kant [8] using the HSDT. The static behavior of laminated composite and sandwich plates are solved analytically using Navier’s technique based on higher order refined theory by Kant and Swaminathan [9]. Zhang and Kim [10] presented geometrically nonlinear static responses of laminated composite plates by using two new displacement based quadrilateral plate (RDKQ-NL20 and RDKQ-NL24) elements. Soltani *et al*. [11] analyzed nonlinear tensile behavior of glass fibre reinforced aluminium laminates using finite element modeling approach under in-plane loading. Alibeigloo and Shakeri [12] reported 3-D solution for static analysis of laminated cylindrical panel using differential quadrature method (DQM). Akavci *et al.* [13] investigated the symmetrically laminated composite plates on elastic foundation in the framework of the FSDT. Attallah *et al*. [14] presented 3-D solutions of laminated composite plates by using a combined finite strip and state space approach. Bhar *et al*. [15] analyzed the significance of the HSDT over the FSDT for laminated composite stiffened plates by using FEM. Lu *et al*. [16] studied the semi analytical solutions for bending and free vibration of laminated composite plates using three dimensional elasticity theories, which perfectly combine the state space approach and the technique of differential quadrature. Moleiro *et al*. [17] investigated static and free vibration analysis of laminated composite plates by developing a new mixed least square finite element models in the framework of the FSDT. Casrellazzi *et al*. [18] studied nodally integrated plate element formulation for the analysis of laminated composite plates based on the FSDT. Grover *et al*. [19] investigated the static and buckling analysis of laminated composite and sandwich plates in the framework of a new inverse hyperbolic shear deformation theory and this theory is based on shear strain shape function. Refined laminated composite plate element based on the global local HSDT is analyzed by wu *et al*. [20] and this theory satisfies fully the free surface conditions and the geometries and stress continuity conditions at interfaces. Ren [21] developed a new theory of laminated plate and the equilibrium equations and boundary conditions are similar to those of the classical plate theory. Krishna Murty and Vellaichamy [22] studied the suitability of the HSDT for the stress analysis of laminated composite panels based on cubic in plane displacements and parabolic normal displacements. Ray [23] developed a zeroth order shear deformation theory for the static and dynamic analysis of laminated composite plates. Nonlinear static behavior of fibre reinforced plastic (FRP) laminates with circular cutout on the effect of thickness ratio and skew angle are analyzed by Raju *et al*. [24] and [25], respectively. Pradyumna and Bandyopadhyay [26] investigated the static and free vibration behavior of laminated shells using a higher order theory and Sander's approximation considering the effect of rotary inertia and transverse shear. Kumar *et al*. [27] studied static analysis of thick skew laminated composite plate with elliptical cutout based on 3-D elasticity theories. Riyah and Ahmed [28] investigated stresses of composite plates with different types of cutouts. Rezaeepazhand and Jafari [29] analytically analyzed the stresses of composite plates with a quasi-square cutout subjected to uniaxial tension based on Lekhnitskii's theory. Upadhyay and Shukla [30] investigated the large deformation flexural responses of composite laminated skew plates based on third order shear deformation theory (TSDT) and von-Karman's nonlinearity. Static analysis of isotropic rectangular plate with various support conditions and loads are studied by Vanam *et al*. [31] using finite element analysis (FEA).

2.3 Vibration analysis of laminated composite structures

The structures are exposed to dynamic type of loading during their service and which may in turn the structure to vibrate. Kant and Swaminathan [32] analyzed analytically the free vibration responses of laminated composite and sandwich plates based on a higher order refined theory and used Navier's technique to obtain the solution in closed form. Khdeir and Reddy [33] studied free vibration behavior of laminated composite plates using second order shear deformation theory and a generalized Levy type solution in conjunction with the state space concept. Reddy and Liu [34] presented Navier type exact solutions for bending and natural vibrations of laminated elastic cylindrical and spherical shells based on the HSDT. Thai and Kim [35] examined the free vibration responses of laminated composite plates using two variables refined plate theory. Reddy [36] studied the free vibration of anti-symmetric angle ply laminated plates including transverse shear deformation using FEM. Ganapathi *et al*. [37] analyzed the free vibration analysis of simply supported composite laminated panels in the framework of FSDT and obtained governing equations using energy method. Swaminathan and Patil [38] analyzed analytically the free vibration responses of anti-symmetric angle ply plates using a higher order refined computational model with twelve degree of freedom. Bhimaraddi [39] predicted the fundamental frequency of laminated rectangular plates using Kirchhoff plate theory and parabolic shear deformation theory. Chakravorty *et al*. [40] presented a finite element analysis of the free vibration behavior of point supported laminated composite doubly curved shells in the framework of the FSDT. Luccioni and Dong [41] reported Levy type semi analytical solutions of free vibration and stability behavior of thin and thick laminated composite rectangular plates based on the CLPT and the FSDT. Xiang *et al*. [42] reported free vibration behavior of laminated composite plates based on the n^{th} order shear deformation theory and this theory satisfies the zero transverse shear stress boundary conditions. Putcha and Reddy [43] developed a mixed shear flexible finite element model to analyze geometrically linear and nonlinear stability and free vibration behavior of layered anisotropic plates based on the refined higher order theory. Reddy and Phan [44] reported exact solutions of stability and vibration responses of isotropic and orthotropic simply supported plates according to the HSDT. Reddy and Kuppusamy [45] reported 3-D elasticity solutions natural vibrations of laminated anisotropic plates. Liew *et al*. [46] employed the moving least squares differential quadrature method for vibration analysis of symmetrically laminated plates based on FSDT. Khdeir [47] investigated the free vibration of anti-symmetric angle ply laminated plates based on a generalized Levy type solution. This theory is a generalization of Mindlin's theory for isotropic plates to laminated anisotropic plates. Ferreira *et al*. [48] employed the FSDT in the multi quadric radial basis function for predicting the free vibration behavior of moderately thick symmetrically laminated composite plates. Kant and Swaminathan [49] studied the free vibration behavior of isotropic, orthotropic and multilayer plates based on higher order refined theory. Srinivas *et al*. [50] employed a three dimensional linear, small deformation theory for the free vibration analysis of simply supported homogeneous and laminated thick rectangular plates. Pandit *et al*. [51] studied the free undamped vibration of isotropic and fibre reinforced laminated composite plates in the framework of FSDT and recommended an effective mass lumping scheme with rotary inertia. Ovesy and Fazilati [52] employed the third order shear deformation theory for buckling and free vibration finite strip analysis of composite plates with cutout based on two different modeling approaches (semi analytical and spline method). Reddy [53] studied the large amplitude flexural vibration of layered composite plates with cutout based on a Reissner-Mindlin type of a shear deformable theory and employed the nonlinear strain displacement relations of the von-Karman theory. Sivakumar *et al*. [54] investigated the free vibration responses of composite plates with an elliptical cutout based on FSDT and using a genetic algorithm. Kumar and Shrivastava [55] employed a finite element formulation based on HSDT and Hamilton's principle to study the free vibration responses of thick square composite plates having a central rectangular cutout, with and without the presence of a delamination around the cutout. Sahu and Datta [56] studied the dynamic stability of curved panels with cutouts in the framework of FSDT and used the Bolotin's method. Ju *et al*. [57] employed a finite element approach to analyze the free vibration behavior of square and circular composite plates with delaminations around internal cutouts. Boay [58] analyzed the free vibration responses of laminated composite plates with a central circular hole. Lee *et al*. [59] employed a simple numerical method based on the Rayleigh principle for predicting the natural frequencies of composite rectangular plates with rectangular cutouts. Liew *et al*. [60] analyzed the free vibration of rectangular plates with internal discontinuities due to central cutouts using the discrete Ritz method. Dhanunjaya Rao and Sivaji Babu [61] studied the modal analysis of thin FRP skew symmetric angle ply laminate with circular cutout in the framework of CLT. Krishna Reddy and Palaninathan [62] analyzed the free vibration responses of laminated skew plates using a general high precision triangular plate bending finite element. Garg *et al.* [63] employed a simple C^0 isoparametric finite element model to predict the free vibration responses of isotropic, orthotropic and layered anisotropic composite and sandwich skew laminates based on HSDT. The large amplitude free flexural vibration behaviors of thin laminated composite skew plates are investigated by Singha and Ganapathi [64] using finite element approach. Wang [65] employed a B-spline Rayleigh-Ritz method to predict the free vibration responses of skew isotropic plates and fibre reinforced composite laminates based on FSDT.

2.4 Aim and scope of present study

Based on the literature review it is seen that the study of skewed plate with cutout using commercial software are limited in number. Commercial software ANSYS is well accepted in many industries because of less computational cost with good accuracy. Hence, the aim of present study is to develop a FEM model using APDL code in ANSYS environment for skewed laminated plate with cutout. The study is extended to analyze the vibration and bending behavior of laminated plate. The effect of different geometries of cutout and skew angles on the different responses will be evaluated based on developed model.

3. THEORETICAL FORMULATION

3.1 Finite Element Modeling

The FEM is the preeminent discretization technique in structural mechanics. The basic concept of the FEM is that, subdivision of the mathematical model into disjoint components through shape function called finite elements in the physical interpretation. A finite number of degree of freedom is expressed in terms of an unknown function.

For the modeling purpose, a SHELL181 element is being selected among the available elements in ANSYS 14.0 element library. Fig. 1 shows solid geometry, node locations and the element coordinate system of the SHELL181 element. The element is defined by four nodes (I, J, K and L). This is a 2-D four noded element with six degree of freedom per node and the degree of freedoms of each node are the translations and rotations in the respective axis. This element has the capability to analyze layered applications such as composite and sandwich structures. It is suitable for thin to moderately thick structures and well suited for linear, large rotation and large strain nonlinear applications.

It is well known that the mathematical model in ANSYS is based on the FSDT as follows:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) \\ w(x,y,z) &= w_0(x,y) + z\theta_z(x,y) \end{aligned} \tag{1}$$

where, *u*, *v* and *w* represents the displacements of any point along the (*x*, *y*, *z*) coordinates. u_0, v_0 are the in-plane and w_0 is the transverse displacements of the mid-plane and θ_x, θ_y are the rotations of the normal to the mid plane about *y* and *x* axes respectively and θ_z is the higher order terms in Taylor's series expansion. The geometry of two dimensional laminated composite plates with positive set of coordinate axis and the middle plane displacement terms are shown in Fig. 2 and 3, respectively. The schematic diagram of a composite plate and skew plate with and without cutout is shown in Fig. 4. where, α denotes the skew angle of the laminated plate. Fig. 5

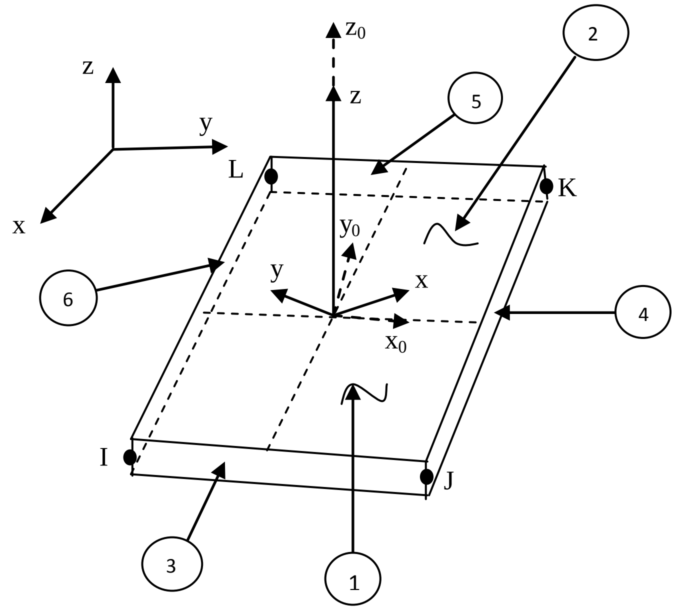

Fig. 1 Geometry of SHELL181 element

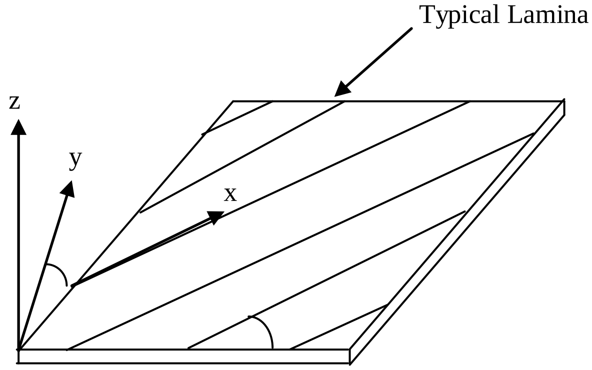

Fig. 2 Typical presentation of general lamina geometry of a laminated composite plate with an orientation of fibre

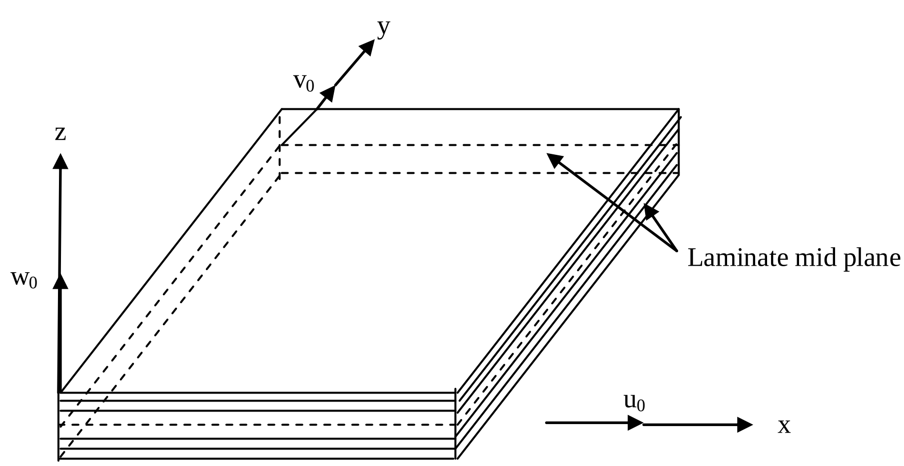

Fig. 3 Laminate geometry with positive set of laminate reference axis

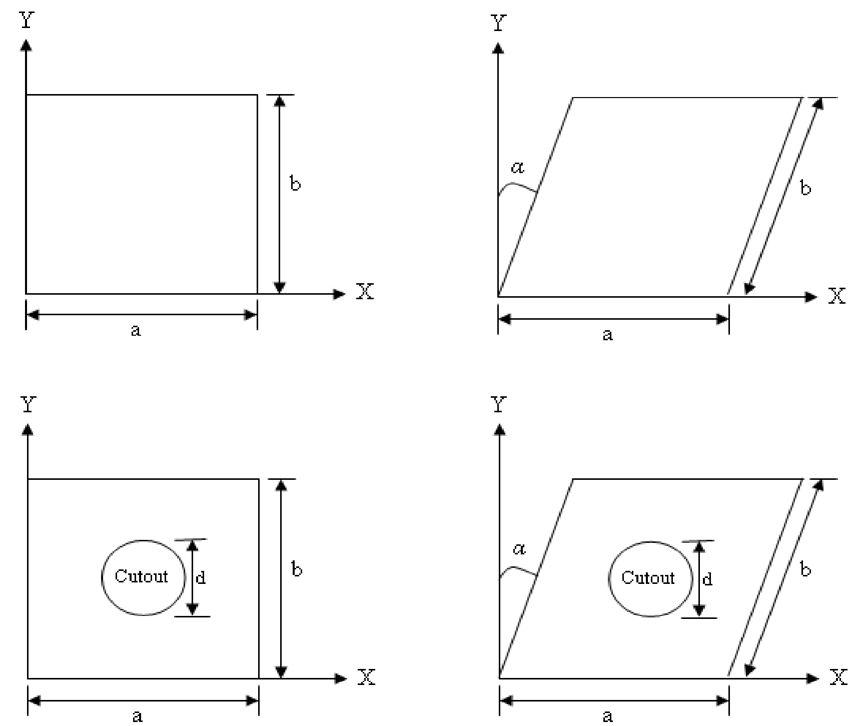

Fig. 4 Representation of laminated plate and skew plate with and without cutout

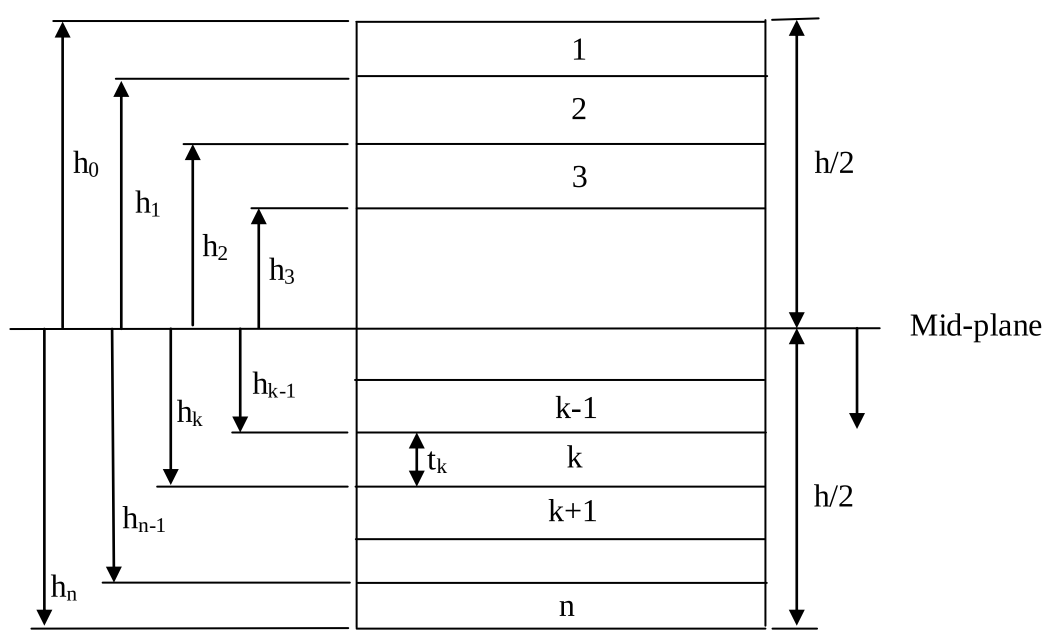

Fig. 5 Coordinate locations of plies in a laminate

shows a laminate made of n plies and each ply has a thickness of t_k. An ANSYS model of the same plate has been developed and presented in Fig. 6.

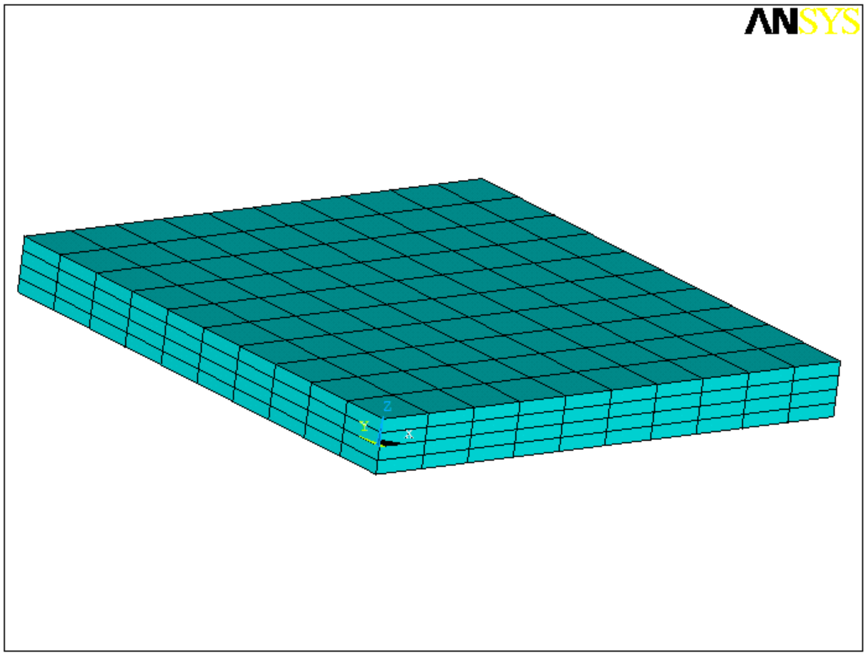

Fig. 6 Laminated composite plate modeled in ANSYS

Strains are obtained by derivation of displacements as:

$$\{\varepsilon\} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial w}{\partial z} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \\ \dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y} \\ \dfrac{\partial w}{\partial x} + \dfrac{\partial u}{\partial z} \end{Bmatrix} \tag{2}$$

where, $\{\varepsilon\} = \{\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \gamma_{xy} \quad \gamma_{yz} \quad \gamma_{xz}\}^T$, is the normal and shear strain components of in plane and out of plane direction.

The strain components are now rearranged in the following steps.

The in-plane strain vector:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{x_0} \\ \varepsilon_{y_0} \\ \gamma_{xy_0} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{3}$$

The transverse strain vector:

$$\begin{Bmatrix} \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{z_0} \\ \gamma_{yz_0} \\ \gamma_{xz_0} \end{Bmatrix} + z \begin{Bmatrix} \kappa_z \\ \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} \tag{4}$$

where, the deformation components are described as:

$$\begin{Bmatrix} \varepsilon_{x_0} \\ \varepsilon_{y_0} \\ \gamma_{xy_0} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} ; \quad \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \theta_x}{\partial x} \\ \dfrac{\partial \theta_y}{\partial y} \\ \dfrac{\partial \theta_x}{\partial y} + \dfrac{\partial \theta_y}{\partial x} \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} \varepsilon_{z_0} \\ \gamma_{yz_0} \\ \gamma_{xz_0} \end{Bmatrix} = \begin{Bmatrix} \theta_z \\ \dfrac{\partial w_0}{\partial y} + \theta_y \\ \dfrac{\partial w_0}{\partial x} + \theta_x \end{Bmatrix} ; \quad \begin{Bmatrix} \kappa_z \\ \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \dfrac{\partial \theta_z}{\partial y} \\ \dfrac{\partial \theta_z}{\partial x} \end{Bmatrix} \tag{6}$$

The strain vector expressed in terms of nodal displacement vector:

$$\{\varepsilon\} = [B]\{\delta\} \tag{7}$$

where, $[B]$ is the strain displacement matrix containing interpolation functions and their derivatives and $\{\delta\} = \{u_0 \quad v_0 \quad w_0 \quad \theta_x \quad \theta_y \quad \theta_z\}^T$, is the nodal displacement vector.

For a laminate, the generalized stress strain relationship with respect to its reference plane may be expressed as:

$$\{\sigma\} = [D]\{\varepsilon\} \tag{8}$$

where, $\{\sigma\}$ and $\{\varepsilon\}$ is the stress and strain vector respectively and $[D]$ is the rigidity matrix.

The strain displacement matrix $[B]$ is given by,

$$[B] = \sum_{i=1}^{4} \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & 1 & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial x} & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial x} \end{bmatrix} \tag{9}$$

3.2 Plate Element Formulation

The displacement components of the four noded isoparametric elements are

$$\{\delta\} = \{u_0 \quad v_0 \quad w_0 \quad \theta_x \quad \theta_y \quad \theta_z\}^T \tag{10}$$

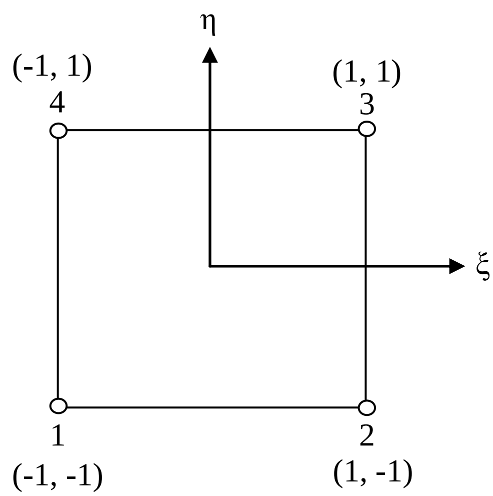

Fig. 7 Isoparametric quadratic shell element

The element displacement are expressed in terms of their nodal values by using the element shape function and is given by,

$$u = \sum_{i=1}^{4} N_i u_i \qquad v = \sum_{i=1}^{4} N_i v_i \qquad w = \sum_{i=1}^{4} N_i w_i$$

$$\theta_x = \sum_{i=1}^{4} N_i \theta_{x_i} \qquad \theta_y = \sum_{i=1}^{4} N_i \theta_{y_i} \qquad \theta_z = \sum_{i=1}^{4} N_i \theta_{z_i}$$

The shape function at node *i* shown in Fig. 7 is given by,

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

The generalized strain components are,

$$\{\varepsilon\} = \sum_{i=1}^{4} [B]_i \{\delta\}_i \tag{11}$$

By using the chain rule,

$$\frac{\partial}{\partial \xi} = \frac{\partial}{\partial x} * \frac{\partial x}{\partial \xi} + \frac{\partial}{\partial y} * \frac{\partial y}{\partial \xi} \tag{12}$$

$$\frac{\partial}{\partial \eta} = \frac{\partial}{\partial x} * \frac{\partial x}{\partial \eta} + \frac{\partial}{\partial y} * \frac{\partial y}{\partial \eta} \tag{13}$$

Rewritten these in matrix form,

$$\begin{Bmatrix} \dfrac{\partial}{\partial \xi} \\ \dfrac{\partial}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial y}{\partial \xi} \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{Bmatrix} \tag{14}$$

Here, in the derivatives the left hand side column vector is called local derivative and the right hand side column vector is called global derivative. The square matrix in this equation is called the Jacobian matrix for the two dimensional domain and is denoted as

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial y}{\partial \xi} \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta} \end{bmatrix} \tag{15}$$

Eq. (14) can be rewritten as

$$\begin{Bmatrix} \dfrac{\partial}{\partial \xi} \\ \dfrac{\partial}{\partial \eta} \end{Bmatrix} = [J] \begin{Bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{Bmatrix} \tag{16}$$

$$\begin{Bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \dfrac{\partial}{\partial \xi} \\ \dfrac{\partial}{\partial \eta} \end{Bmatrix} \tag{17}$$

The element stiffness matrix is derived using the principle of minimum potential energy.

The potential energy of the plate element is given by,

$$\phi = \frac{1}{2}\int\int \{\delta\}^T [B]^T [D][B]\{\delta\}\, dxdy - \int\int \{\delta\}^T [N]^T\, qdxdy \tag{18}$$

where, q is any discrete loading inside the element.

The principle of minimum potential energy requires,

$$\left\{\frac{\partial \phi}{\partial\{\delta\}}\right\} = \{0\}$$

$$\left\{\frac{\partial \phi}{\partial\{\delta\}}\right\} = \int\int [B]^T [D][B]dxdy\{\delta\} - \int\int [N]^T qdxdy = \{0\} \tag{19}$$

$$[K]\{\delta\} = \{P\}$$

where,

[K] = $\int\int [B]^T [D][B]dxdy$

{P} = $\int\int [N]^T qdxdy$

The element stiffness matrix [*K*] and mass matrix [*M*] can expressed as:

$$[K] = \int_{-1}^{+1}\int_{-1}^{+1} [B]^T [D][B]|J|\, d\xi d\eta \tag{20}$$

$$[M] = \int_{-1}^{+1}\int_{-1}^{+1} [N]^T [m][N]|J|\, d\xi d\eta \tag{21}$$

where, $|J|$ is the determinant of the Jacobian matrix, $[N]$ is the shape function matrix and $[m]$ is the inertia matrix. The integration has been carried out using the Gaussian quadrature method.

The static analysis determines the deflections as:

$$[K]\{\delta\} = \{P\} \tag{22}$$

where, $\{P\}$ is the static load vector acting at the nodes.

The free vibration analysis involves determination of natural frequencies as:

$$\left([K] - \omega_n^2 [M]\right) = 0 \tag{23}$$

The above governing Eq. (22) and (23) are solved imposing the boundary condition and the solution algorithm.

3.3 Boundary condition

The purpose of boundary condition in any solution is to avoid the rigid body motion and to get the responses by reducing the number of field variables. In order to solve the governing equations as discussed in the aforementioned line are solved using different support conditions. The supports conditions are discussed mathematically as well as a schematic presentation have been given in Fig. 8. where, S and C stand simply supported and clamped condition, respectively.

Clamped on all edges: BC-I

$u = v = w = \theta_x = \theta_y = \theta_z = 0$, at $x = 0, a$ and $y = 0, b$.

Simply supported on all edges: BC-II

SS1: $u = w = \theta_x = 0$, at $x = 0, a$;

$v = w = \theta_y = 0$, at $y = 0, b$.

SS2: $u = w = \theta_y = 0$, at $x = 0, a$;

$v = w = \theta_x = 0$, at $y = 0,b$.

SS3: $v = w = \theta_x = 0$, at $x = 0, a$;

$u = w = \theta_y = 0$, at $y = 0, b$.

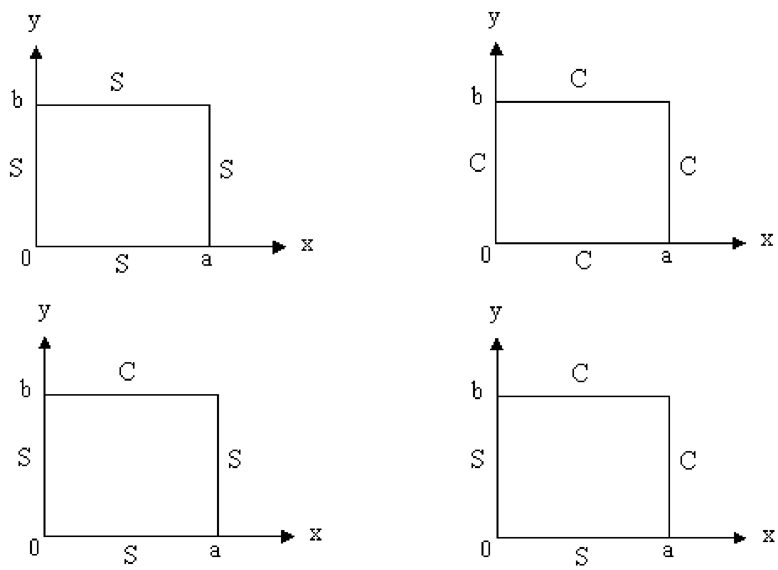

Fig. 8 Representation of different support condition for the analysis

3.4 Solution technique and steps

The equations are solved using two different algorithms to obtain the vibration and static responses such as Block Lanczos and Gauss elimination method, respectively. The solution steps are given in detail in reference [7]. The steps have been followed for the solutions are given in the Fig. 9.

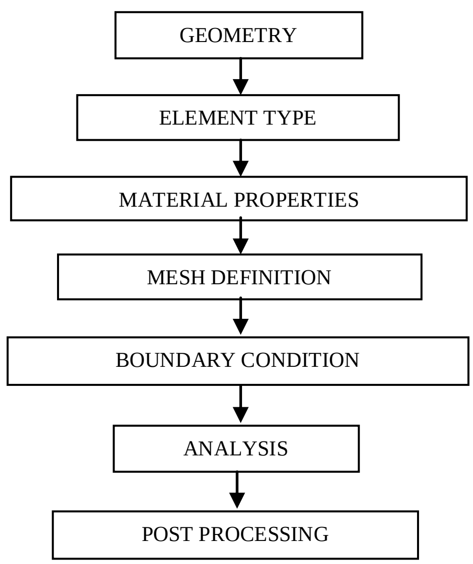

Fig. 9 Solution steps in ANSYS

4. RESULTS AND DISCUSSION

The composite plates/skew plates with arbitrary geometries and boundary conditions are widely used as structural elements in aerospace, civil, mechanical and other structures. These structures generally under severe dynamic loading and different constrained conditions during their service life. Hence, for the designer's quest to model these complex structural problems precisely and predict the deflections, natural frequencies and other responses with less computational effort.

The proposed model has been developed based on the finite element steps as in ANSYS and solved using the APDL coding. In order to prove the efficacy of the present model, convergence test has been done. Based on the convergence, static and vibration responses are obtained and compared with those published results. The influences of various parameters on the static and free vibration analysis of laminated composite plates/skew plates are generated by solving some new examples.

4.1 Material properties

The material properties considered for the present numerical analysis are given in Table 1 and Table 2.

Table 1 Elastic property for static analysis.

Material	E_1	E_2	E_3	ν_{12}	ν_{23}	ν_{13}	G_{12}	G_{23}	G_{13}
1 [10]	12.605	12.628	12.628	0.23949	0.23949	0.23949	2.154	2.154	2.154
2 [25]	141.68	12.384	12.384	0.25772	0.42057	0.25772	3.88	4.36	3.88
3 [30]	10	1	1	0.22	0.22	0.22	$0.33E_2$	$0.2E_2$	$0.33E_2$

The units of E_1, E_2, E_3, G_{12}, G_{23} and G_{13} of material 1 and 2 are in *GPa* and the properties of material 3 are normalized by E_2.

Table 2 Elastic properties for free vibration analysis.

Material	E_1	E_2	ν_{12}	ν_{23}	ν_{13}	G_{12}	G_{23}	G_{13}	ρ(kg/m^3)
4 [34]	25	1	0.25	0.25	0.25	$0.5E_2$	$0.2\,E_2$	$0.5\,E_2$	1
5 [54]	40	1	0.25	0.25	0.25	$0.6\,E_2$	$0.5\,E_2$	$0.6\,E_2$	1500
6 [63]	40	1	0.25	0.25	0.25	$0.6\,E_2$	$0.5\,E_2$	$0.5\,E_2$	1
7 [61]	147	10.3	0.27	0.54	0.27	7	3.7	7	1600

The properties of materials 4, 5 and 6 are normalized by E_2 and the units of E_1, E_2, G_{12}, G_{23} and G_{13} of material 7 are in *GPa*.

4.2 Convergence and Validation Study

In this section, the convergence study of static and free vibration analysis of laminated composite plates/skew plates with and without cutout is computed and the results are compared with the available published literature obtained using different numerical methods.

4.2.1 Laminated Composite Plate

4.2.1.1 Static analysis without cutout

In this example, a clamped four layers symmetric cross ply $(0^{\circ}/90^{\circ})_s$ square plate with length 12 in. and thickness 0.096 in. under uniformly distributed load is considered as in the reference. The material properties and support condition are taken as Material 1 (Table 1) and BC-I condition for the numerical analysis. The central deflection obtained using the present model with different mesh sizes are plotted in Fig. 10 and compared with reference [10]. It can be seen easily that the results are showing good agreement with the reference with a very small difference around 2.8%. The present results are showing little bit lower side as compared to the reference i.e., because of the fact that drilling degree of freedom in transverse direction. It is

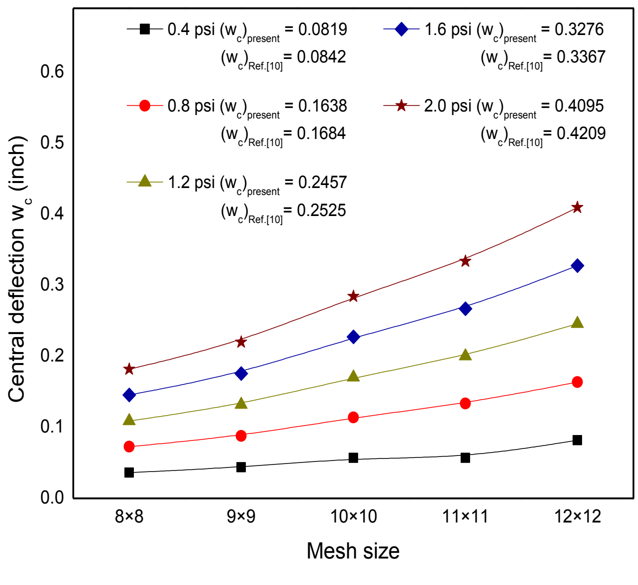

Fig.10 Variation of w_c with distributed load and mesh density.

important that the reference and the present both are based on the FSDT kinematic model and the latter one is more realistic in nature due to an extra rotational term. A (12 × 12) mesh has been considered for the further computation of static behavior of laminated structures without cutout.

4.2.1.2 Static analysis with cutout

A clamped four layers symmetric cross ply (0˚/90˚) $_s$ square plate with circular cutout at centre with side length of 20 mm and the total thickness is 0.5 mm. The transverse pressure is applied on top surface and size of circular cutout is taken as the ratio of diameter to side length is 0.2. To compare the accuracy of the present developed model, numerical results are evaluated based on the material properties as material 2 (Table 1) and the support is of BC-I type as in the reference. All the layers are assumed to be same density, thickness and made of same material properties. The deflections obtained from different transverse pressure and mesh size are plotted in Fig. 11 and compared with reference [25]. A good convergence rate with mesh refinement can be seen as well as the difference between results are within 7.5%. The difference between the results are reduces with increase in load value. Though the reference and present both are solving using ANSYS, there is a considerable difference exist between this two i.e., because the reference

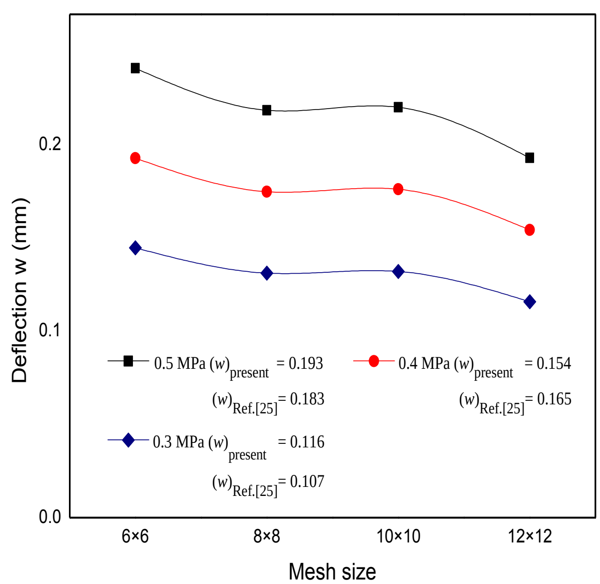

Fig.11 Variation of *w* with transverse pressure and mesh density.

used a 3D brick element (SOLID95) whereas the present analysis used a 2D element (SHELL181). However, the present results are showing a very good convergence and small difference with the published model, developed mathematically for static analysis with cutout. In addition to that, the 3D elements need to be volume mesh, model generation is difficult and the computational time is high as compared to the present. Hence, the present results are well accepted within the range as both is analyzed using the same tool regarding model development and computational cost. A (12 × 12) mesh has been used for further computation results of laminated composite with cutout.

4.2.1.3 Vibration analysis without cutout

In this problem, simply supported cross ply [(0˚/90˚/0˚) and (0˚/90˚) $_s$] square plate is being analyzed for two different thickness ratios (*a*/*h*) 10 and 100 and the responses are presented in Fig. 12 and Fig. 13. For the numerical computation the material properties and boundary condition are taken as material 4 (Table 2) and SS1 type of support from BC-II condition. The figure shows the rate of convergence and comparison study with the published reference [34]. It can be seen that the moderately thick plate (*a*/*h*=10) results are converging at a (30 × 30) mesh and for thin plate (*a*/*h*=100), a (26 × 26) mesh is giving good convergence rate. The results are showing negligible difference for thin plates with the analytical solution whereas for thick plates the difference is not major. In addition to that it will be important to mention the

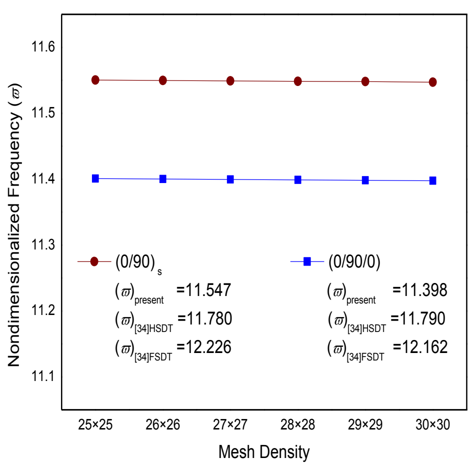

Fig.12 Variation of $\bar{\omega}$ with stacking sequence and mesh density for *a*/*h*=10.

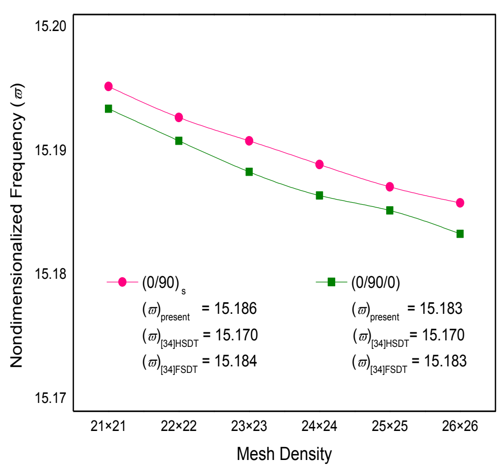

Fig.13 Variation of $\bar{\omega}$ with stacking sequence and mesh density for *a*/*h*=100.

model has been developed in the framework of HSDT in the reference whereas the present one the FSDT. The results show the accuracy of the developed model with little effort, l computational cost and time with unmatched precision.

If not stated otherwise, the fundamental frequencies are nondimensionalised for vibration of laminated plate without cutout in throughout analysis as follows:

$$\bar{\omega} = \omega a^2 / h \sqrt{\rho / E_2}.$$

4.2.1.4 Vibration analysis with cutout

A simply supported cross ply (0˚/90˚) square plate with square cutout at centre for different thickness ratios (*a*/*h* = 5, 10, 20, 25, 100) is taken for the computation purpose. In this analysis, material properties are takes as material 5 (Table 2) and support condition of SS1 type from BC-II. The nondimensional frequencies ($\bar{\omega} = \omega a^2 \sqrt{\rho h / D}$) of laminated plate with square central cutout, *c*/*a*=0.5 (where, *c* and *a* are the side length of cutout to plate respectively) having different thickness ratios are shown in Table 3 and compared with the references [52] and [53]. It can be seen that the present results are showing negligible difference with former reference as compared to the latter. This is because of the fact the former model is developed based on the HSDT and a very recent work whereas the latter is formulated based on the FSDT kinematic model. We note that the differences are reduces as the thickness ratio increases in most of the cases. Based on the convergence a (25 × 25) mesh is used for the computation of new results. The nondimensional fundamental frequencies of laminated composite plate with different cutout sizes are obtained further using the same mathematical formulae as discussed in the above lines.

Table 3 Nondimensional frequency ($\bar{\omega} = \omega a^2 \sqrt{\rho h / D}$) for square plate with square central cutout having different thickness ratios. (App. 1: Longitudinal strip assemble, App. 2: Negative stiffness)

Source	*a*/*h*				
	5	10	20	25	100
Present	33.571	41.351	46.0855	47.133	50.531
Ref. [52] (App.1)	34.682	42.748	46.580	47.179	48.415
(App.2)	34.778	42.882	46.738	47.340	48.587
Ref. [53]	36.583	43.728	46.971	47.464	48.414

4.2.2 Laminated Composite Skew Plate

4.2.2.1 Static analysis without cutout

In this problem, a clamped four layers symmetric cross ply $(0^{\circ}/90^{\circ})_s$ square skew plate (α = 45˚) is being analyzed for thickness ratio (*a*/*h*=20) and subjected to different uniform transverse pressure (Q = 100, 200, 300, 400 and 500). The material properties are taken as material 3 (Table 1) and the boundary conditions of BC-I type. The nondimensional deflections of laminated skew plate having different load parameters are shown in Table 4 and compared with the reference [30]. It can be seen that the present results are showing negligible difference with the reference. It is important to mention that the model has been developed based on third order shear deformation theory (TSDT) in the reference and the present one is the FSDT. A (4×4) mesh has been considered for the further computation of new results.

Following nondimensional parameters are used in the analysis are:

$Q = qa^4 / E_2 h^4$; $w_c = w_0 / h$.

Table 4 Nondimensional deflections for square skew plate (α = 45˚) having different load parameters.

Source	*Q*				
	100	200	300	400	500
Present	0.2082	0.4166	0.6248	0.8332	1.0414
Ref. [30]	0.2028	0.4183	0.6329	0.8434	1.0638

4.2.2.2 Vibration analysis without cutout

In this example, a simply supported four layers anti-symmetric angle ply $(\pm 45)_2$ square laminated skew plate for different skew angle (α = 0˚, 15˚, 45˚ and 60˚) and the thickness ratio (*a*/*h*=10) is considered for the computation purpose. For the numerical analysis, material properties are taken as material 6 (Table 2) and the support conditions of SS2 type from BC-II. The nondimensional frequencies ($\bar{\omega} = \omega b^2 / \pi^2 h \sqrt{\rho / E_2}$) of laminated skew plate obtained from different skew angles and mesh size are plotted in Fig. 14 and compared with the reference [63]. It is clear from the figure that the results are showing good convergence rate with the mesh refinement. The results are showing negligible difference with the FOST and with the HOST are in the acceptable range. Based on the convergence, a (16 × 16) mesh has been used for the further computation of new results for vibration analysis of laminated skew plate without cutout.

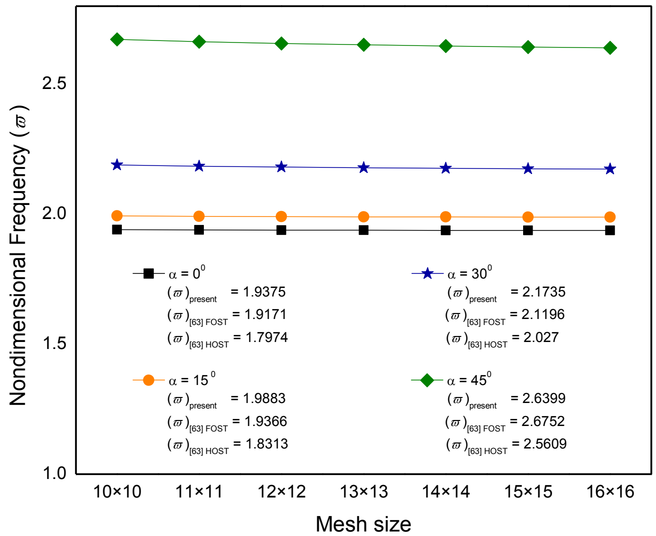

Fig. 14 Variation of $\bar{\omega}$ with different skew angle and mesh size.

4.2.2.3 Vibration analysis with cutout

In this example, a simply supported anti-symmetric angle ply $(\pm 45)_2$ skew plate ($\alpha = 20^\circ$) with circular cutout of diameter 0.2m at center and the thickness ratio (a/h) 50 is considered and

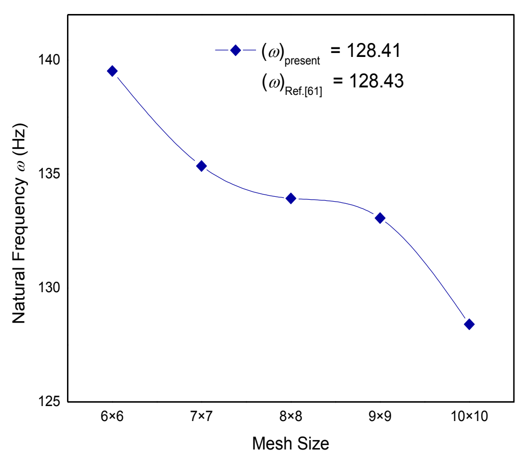

Fig. 15 Variation of ω for anti-symmetric angle ply skew plate with mesh density.

the responses are plotted in Fig. 15. For the computation purpose, material properties are taken as material 7 (Table 2) and the boundary conditions of SS3 type from BC-II. The figure shows the convergence rate with the mesh refinement and compare with the reference [61]. The result obtained using the present model is showing negligible difference with the reference. It is also to mention that present and reference both are solving using ANSYS but model is based on classical lamination theory (CLT) in the reference and the present one is FSDT. For the computation of new results, a (10 × 10) mesh is used.

4.3 Parametric study

In order to build up the confidence some new results are computed using the developed model. A set of parametric study and their effects on static and vibration behavior are discussed in detail. The static and vibration responses of laminated plates/skew plates with and without cutout are subdivided into two subsections for the sake of clarity. The results are computed for different parameters such as lamination schemes, thickness ratio, modular ratios, load, support conditions, cutout geometries, cutout sizes and skew angles.

4.3.1 Laminated Composite Plate

In this section, the static and vibration responses of laminated composite plate with and without cutout are discussed.

4.3.1.1 Static analysis with and without cutout

This section discusses the static behavior of laminated plate with and without cutout for different parameters. The first two paragraphs discusses the plate without cutout (effect of load and lamination schemes) whereas the last three regarding the plate with cutout (effect of load, cutout geometry, lamination scheme and cutout shape).

A clamped laminated square plate subjected to five uniformly distributed load (50 psi, 100 psi, 150 psi, 200 psi and 250 psi) is analyzed for four lamination schemes [(0˚/90˚), (±45˚), (15˚/-30˚) and (-30˚/90˚)]. The responses like, stresses, deflection and deformation shapes of the plate have been presented by taking the plate geometries are12 in. length and 0.096 in. thickness.

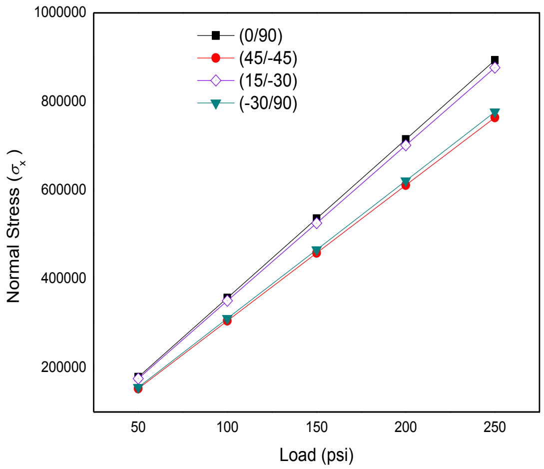

Fig.16 Variation of σ_x with distributed load and lamination scheme.

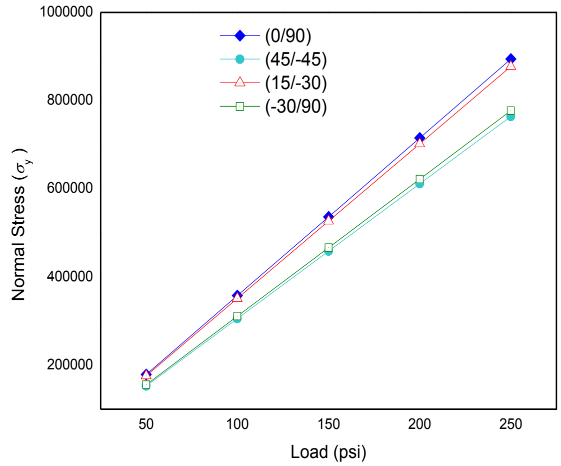

Fig.17 Variation of σ_y with distributed load and lamination scheme.

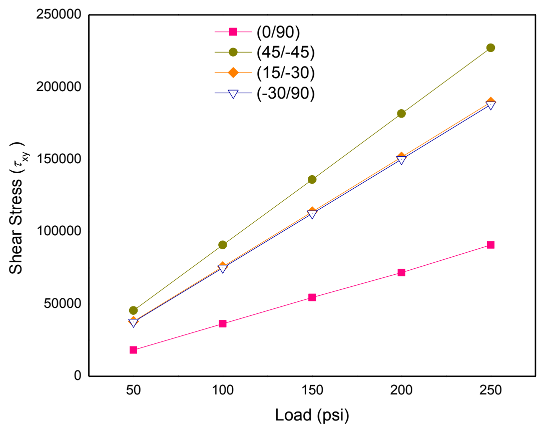

Fig.18 Variation of τ_{xy} with distributed load and lamination scheme.

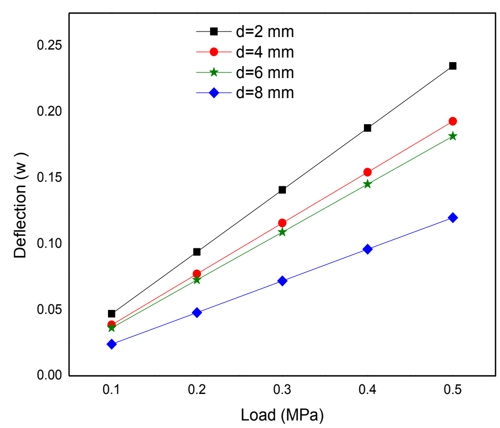

Fig.21 Variation of w with transverse pressure and cutout geometry.

The normal stress in x and y direction and shear stress in x-y plane for four different lamination schemes under uniformly distributed load and clamped conditions are plotted in Fig 16-18. It can be conceded from the figure that the cross ply lamination scheme (0˚/90˚) is showing highest normal stress in x and y direction and the angle ply (±45˚) has the lowest one but the stress value shows an opposite trend in case of shear. The other two lamination scheme is showing an intermediate value for both of the stresses. The results are within the expected line

Table 5 Deflection of different lamination scheme in clamped condition.

Ply Orientation	Load (psi)				
	50	100	150	200	250
(±45˚)	10.439	20.878	31.317	41.756	52.195
(15˚/-30˚)	10.394	20.788	31.183	41.576	51.971
(-30˚/90˚)	10.348	20.696	31.044	41.392	51.739
(0˚/90˚)	10.239	20.477	30.716	40.955	51.194

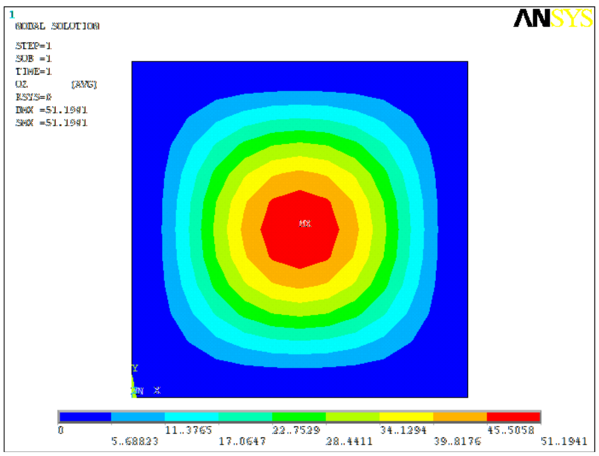

Fig. 19 Deformation shape of cross ply (0˚/90˚) laminate.

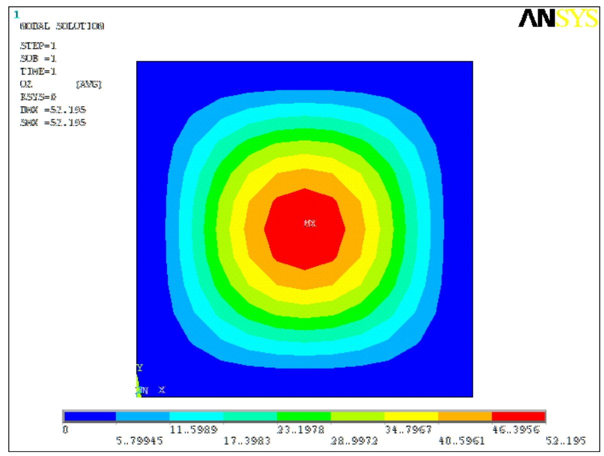

Fig. 20 Deformation shape of angle ply (±45˚) laminate.

for any composite. The deflections are presented in Table 5 and the values are showing in descending order. The deformation shapes for maximum load are generated from ANSYS for two different laminations [(0˚/90˚) and (±45˚)] are shown in Fig. 19 and Fig. 20.

Fig. 21 represent the deflections of a 4 layer clamped symmetric cross ply $(0˚/90˚)_s$ laminate for five transverse pressures (0.1 MPa, 0.2 MPa, 0.3 MPa, 0.4 MPa and 0.5 MPa) and four circular cutout diameter (2 mm, 4 mm, 6 mm and 8 mm). It can be seen that the deflections decreases as the cutout diameter increases. The normal stress (σ_x and σ_y) and shear stress (τ_{xy}) under five different transverse pressures (10 MPa, 20 MPa, 30 MPa, 40 MPa and 50 MPa) and three lamination schemes [$(0˚/90˚)_s$, $(30˚/-50˚)_s$ and $(\pm45˚)_s$] are plotted in Fig. 22-24 for clamped laminated plate. The geometric parameters of plate are taken as of 20 mm length and 0.5 mm thickness and the diameter of circular cutout is 4 mm for the present computation. The

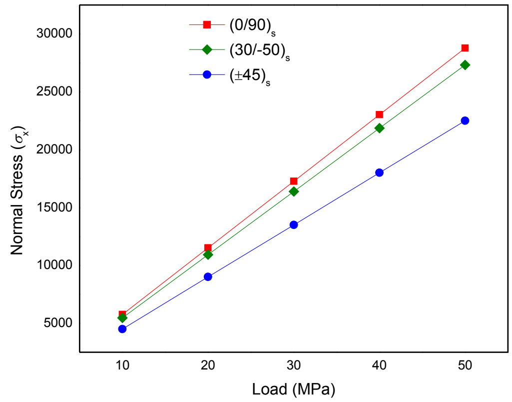

Fig. 22 Variation of σ_x with transverse pressure and lamination scheme.

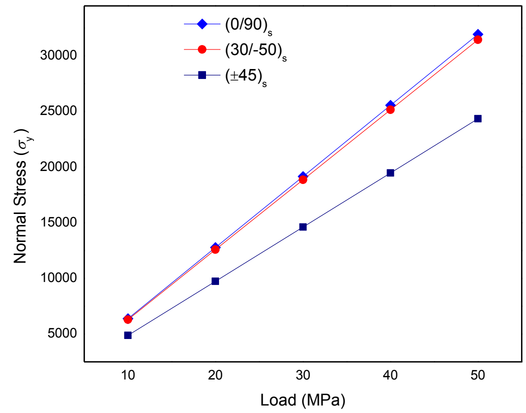

Fig. 23 Variation of σ_y with transverse pressure and lamination scheme.

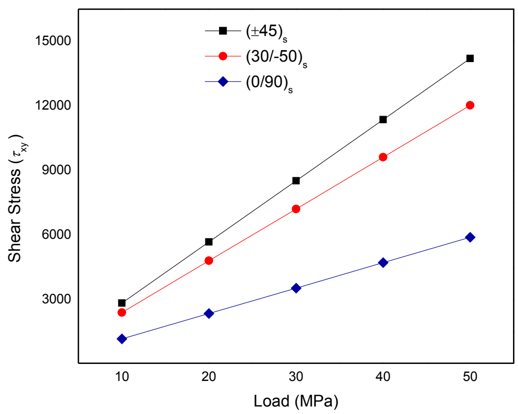

Fig. 24 Variation of τ_{xy} with transverse pressure and lamination scheme.

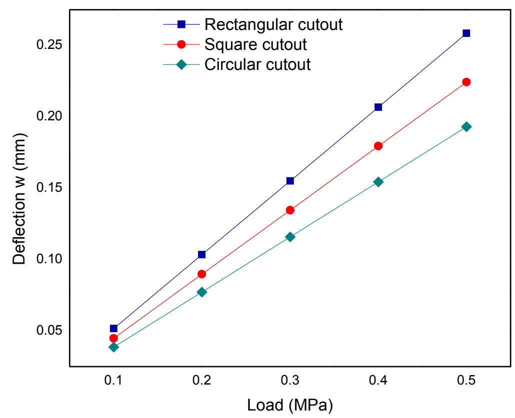

Fig. 25 Variation of w with transverse pressure and cutout shape.

stress (normal stress and shear stress) value follows the same trend as discussed in the laminated plate without cutout.

In addition to the above another problem has been analyzed by considering different cutout geometries (circular, square, rectangular) and loading conditions (0.1 MPa, 0.2 MPa, 0.3 MPa, 0.4 MPa and 0.5 MPa) on deflection behavior of a symmetric cross ply $(0^\circ/90^\circ)_s$ clamped plate is plotted in Fig. 25. The different geometries of the cutout are taken and the geometries are circular cutout of diameter 4 mm, square cutout side of 3.545 mm and rectangular cutout of size 4.189×3 mm. It can be seen that for same cutout area the rectangular cutout and the circular cutout are showing the highest and lowest deflections, respectively. The square cutout is showing an intermediate value.

4.3.1.2 Free vibration analysis with and without cutout

In this section, effects of different parameters on the vibration responses of laminated plate with and without cutout are discussed in detail. The first three paragraphs discusses the plate without cutout (effect of boundary conditions, lamination schemes, modular ratio and no. of layers) whereas the last four regarding the plate with cutout (effect of thickness ratio, modular ratio, boundary condition, no. of layers, cutout ratio, lamination scheme and cutout shape).

The nondimensional frequencies of different mode of a moderately thick laminated plate (a/h = 10) for two cross ply lamination scheme [(0˚/90˚/0˚) and $(0˚/90˚)_s$] at four different support conditions (SSSS, SSSC, SSCC and CCCC) are computed and plotted in Fig. 26 and Fig. 27. From this figure, it can be seen that the frequencies are showing higher and lower value at clamped and simply supported condition, respectively. The other two boundary conditions (SSSC and SSCC) are showing an intermediate value for both lamination schemes.

A new problem has been solved to show the effect of modular ratios on the free vibration behavior. In this analysis a simply supported cross ply moderately thick square plate for two different lamination [(0˚/90˚/0˚) and $(0˚/90˚)_s$] schemes are analyzed and the nondimensional frequencies with five modular ratio (E_1/E_2 = 3, 10, 20, 30 and 40) are plotted in Fig. 28. It can be seen from the figure that the frequencies are increasing as the modular ratio increases but the difference is very small for E_1/E_2, 3 to 10 for both laminations.

It is well known that the composites properties are well dependent on the lamination scheme. Hence, to explore the same an example has been solved here for five anti-symmetric cross ply laminations [(0˚/90˚), $(0˚/90˚)_2$, $(0˚/90˚)_3$, $(0˚/90˚)_4$ and $(0˚/90˚)_5$] simply supported plates. The mode shapes and the nondimensional natural frequencies are plotted in Fig. 29. It can be seen that the natural frequencies of the plate increases with increase in number of layers.

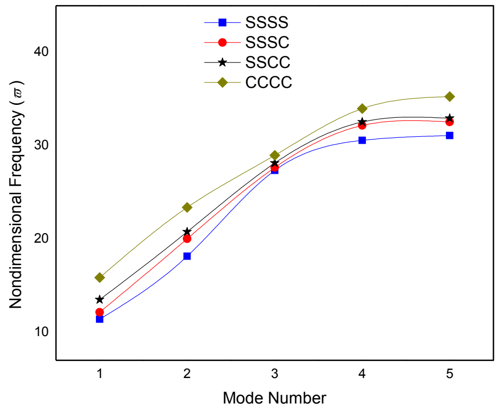

Fig. 26 Variation of $\bar{\varpi}$ with different mode number and boundary conditions for (0˚/90˚/0˚) laminates.

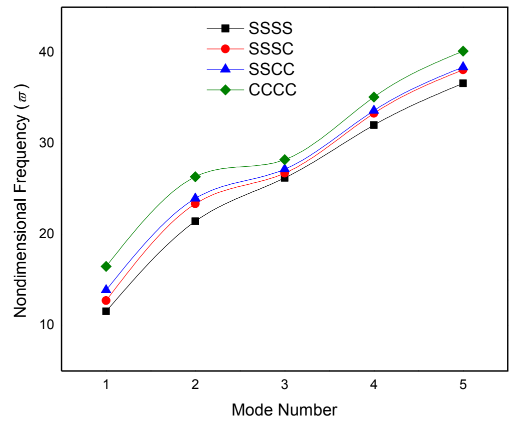

Fig. 27 Variation of $\bar{\varpi}$ with different mode number and boundary conditions for $(0˚/90˚)_s$ laminates.

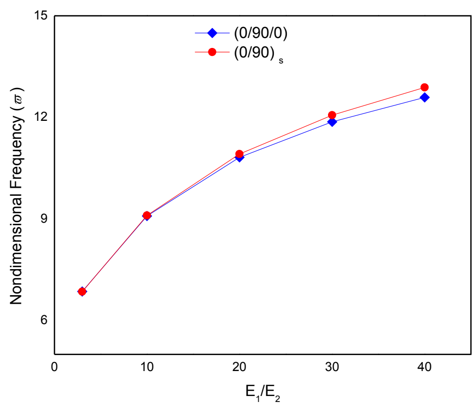

Fig. 28 Variation of $\bar{\varpi}$ with different E_1/E_2 ratios and stacking sequence for a/h=10.

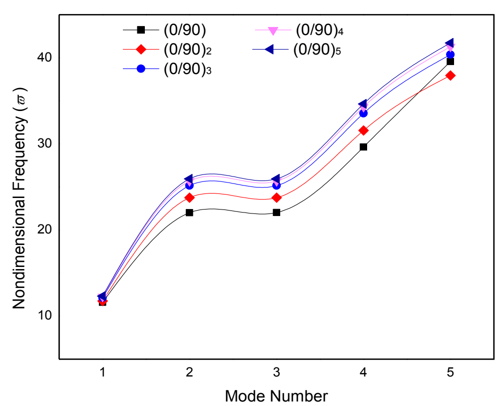

Fig. 29 Variation of $\bar{\varpi}$ with different mode number and no. of plies.

As discussed earlier, cutouts are the structural requirement to meet facilitation as per the need in different structures. In this study analysis has been done for different parameters of laminated plates with cutout. Presently, a simply supported cross ply (0˚/90˚) square plate with square cutout (c/a= 0.5) is analyzed for five different thickness ratio (a/h = 5, 10, 20, 50, 100) and five modular ratios (E_1/E_2 = 3, 10, 20, 30 and 40) and plotted in Fig. 30. It is understood from the figure that the frequency increases for both thickness ratios and the modular ratios. It is because of the fact that as the thickness ratio increases the plate becomes thinner and the

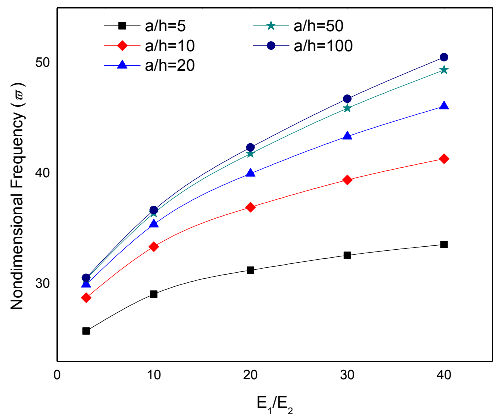

Fig. 30 Variation of $\bar{\omega}$ with different E_1/E_2 and a/h ratio.

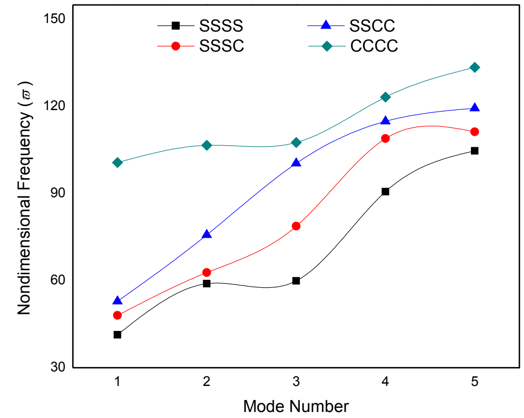

Fig. 31 Variation of $\bar{\omega}$ with different mode number and boundary conditions.

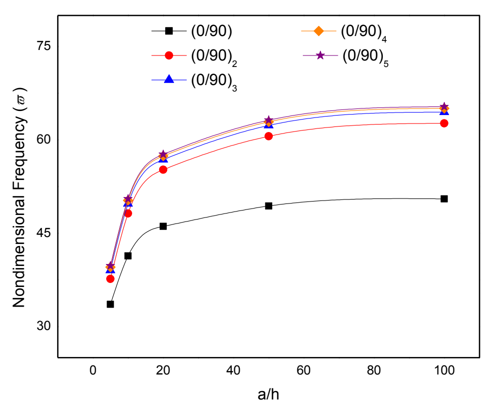

Fig. 32 Variation of $\bar{\omega}$ with different a/h ratio and no. of plies.

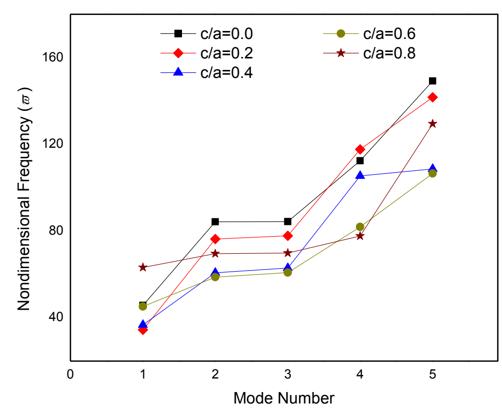

Fig. 33 Variation of $\bar{\omega}$ with different mode number and cutout ratio.

frequency increases. Similarly, the frequency increases as the modular ratio increases as the stiffness is directly proportional to the longitudinal strength.

Effect of different support conditions (SSSS, SSSC, SSCC and CCCC) on the laminated plate with square cutout is analyzed in this example. The nondimensional frequencies of a cross ply (0˚/90˚) moderately thick square plate are shown in Fig. 31. The responses are following the same trend as in case of without cutout case i.e., the frequencies are lower and higher for simply support and clamped case whereas the other two supports are showing the intermediate value.

Another problem of simply supported square plate with cutout has been solved for five different thickness ratios (*a*/*h* = 5, 10, 20, 50 and 100) and five different laminations [(0˚/90˚), $(0˚/90˚)_2$, $(0˚/90˚)_3$, $(0˚/90˚)_4$ and $(0˚/90˚)_5$] and the responses are plotted in Fig. 32. The frequency is showing an increasing trend with the thickness ratios and the number of layers, which is obvious.

Influence of five cutout sizes (*c*/*a*= 0.0, 0.2, 0.4, 0.6 and 0.8) and three cutout geometries (square, rectangular and circular) on vibration behavior of laminated plate of simply supported cross ply (0˚/90˚) square plate is plotted in Fig. 33 and Fig. 34. From the figure it is clear that the values are decreasing as the cutout size increases and the frequencies are increasing with increasing in mode number. Similarly, an uneven behavior can be seen for the latter case as well.

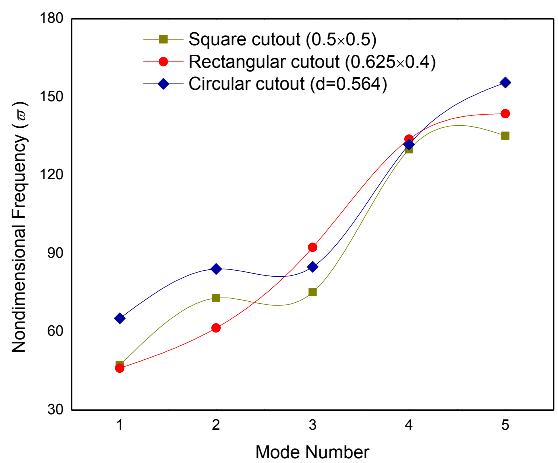

Fig. 34 Variation of $\bar{\omega}$ with different mode number and cutout geometry.

4.3.2 Laminated Composite Skew Plate

As discussed earlier that, some industries are not using regular geometry of composite laminates i.e., the geometry may deviate from basic type (rectangular, square, etc.) and are called skewed. The following section discusses the static and vibration response of the skewed plate.

4.3.2.1 Static analysis with and without cutout

This section discusses the static behavior of laminated skew plate with and without cutout for different parameters. The first four paragraphs discusses the skew plate without cutout (effect of load parameter, modular ratio, thickness ratio, lamination schemes and skew angles) whereas the last four regarding the skew plate with cutout (effect of load, cutout geometry, boundary conditions, lamination scheme and skew angles).

In this example, a clamped laminated symmetric cross ply $(0^\circ/90^\circ)_s$ skew plate is analyzed for thickness ratio (a/h) 20 under four skew angle (α = 15˚, 30˚, 45˚ and 60˚) and five load parameter (Q = 100, 200, 300, 400 and 500) and the nondimensional center deflections are plotted in Fig. 35. It is clear from the figure that the center deflection increases as the load parameter increases and the skew angle decreases.

Another new problem has been solved to show the effect of modular ratio on the static behavior of composite laminated skew plate. In this analysis, a clamped laminated cross ply $(0^\circ/90^\circ)_s$ skew plate with thickness ratio (a/h) 20 and load parameter 100 is analyzed for four skew angles (α = 15˚, 30˚, 45˚ and 60˚) and five modular ratio (E_1/E_2 = 3, 10, 20, 30 and 40) and the nondimensional center deflections are plotted in Fig. 36. It can be seen that the center deflections decreases as the modular ratio and skew angle increases.

The nondimensional center deflections of a clamped laminated cross ply $(0^\circ/90^\circ)_s$ skew plate of load parameter 100 at four different skew angle (α = 15˚, 30˚, 45˚ and 60˚) and five thickness ratio (a/h = 10, 20, 30, 40 and 50) are computed and plotted in Fig. 37. From the figure, it can be seen that the center deflections increases as the thickness ratio increases and the skew angle decreases.

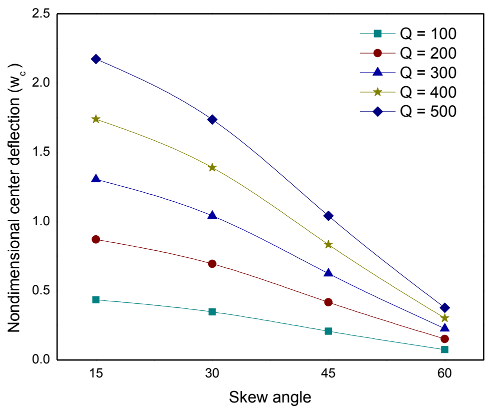

Fig. 35 Variation of w_c with skew angle and load parameter.

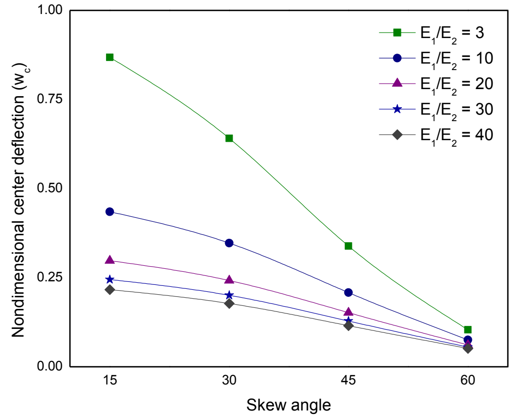

Fig. 36 Variation of w_c with skew angle and modular ratio.

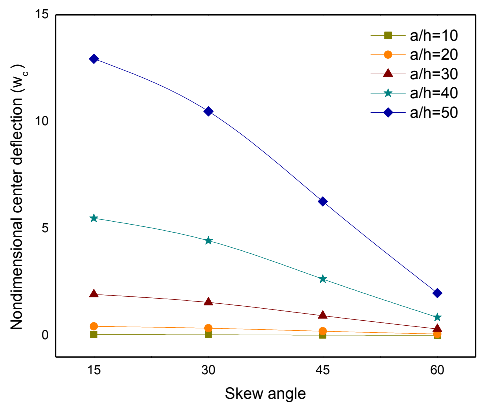

Fig. 37 Variation of w_c with skew angle and thickness ratio.

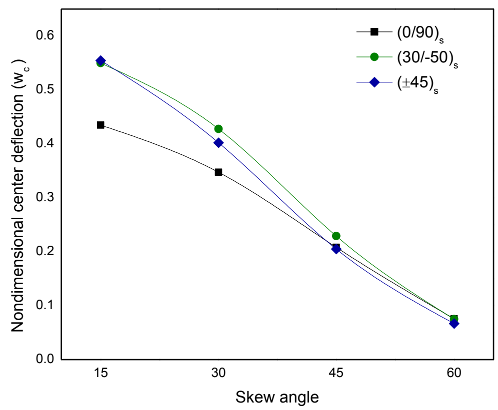

Fig. 38 Variation of w_c with skew angle and ply orientations.

An example has been solved for three different laminations [(0˚/90˚), (30˚/-50˚) and (±45˚)] clamped skew plates for load parameter 100 and thickness ratio 20. The center deflections are plotted in Fig. 38. It can be seen that the center deflections is lower in the case of cross ply and higher in angle ply.

It is well known that cutouts are structural requirements in many industries such as aerospace, civil and mechanical. In this study, a symmetric laminated cross ply $(0˚/90˚)_s$ skew plate having plate geometries are 20 mm length, 0.5 mm thickness and the diameter of circular cutout is 4 mm is analyzed. The deflections of four different skew angle (α = 15˚, 30˚, 45˚ and 60˚) and five transverse pressure (0.1 MPa, 0.2 MPa, 0.3 MPa, 0.4 MPa and 0.5 MPa) is computed and plotted in Fig. 39. It is clear from the figure that the deflection increases as the load increases and skew angle decreases.

Boundary conditions also affect the static behavior of laminated skew plate. In this example, a symmetric cross ply $(0˚/90˚)_s$ laminated skew plate has been analyzed for four

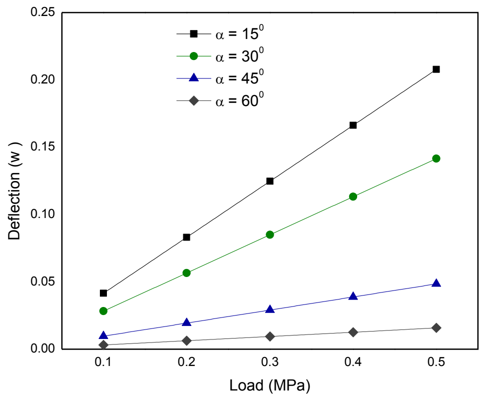

Fig. 39 Variation of *w* with skew angle and transverse load.

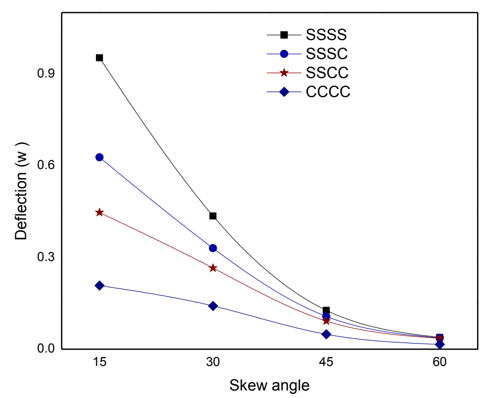

Fig. 40 Variation of *w* with skew angle and boundary conditions.

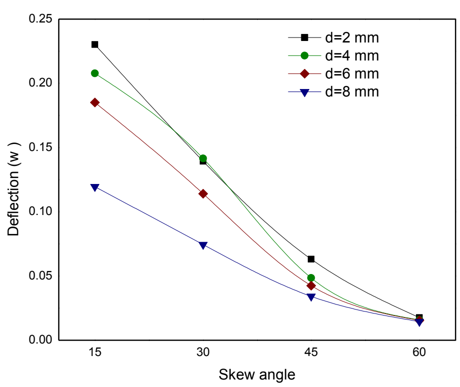

Fig. 41 Variation of *w* with skew angle and cutout size.

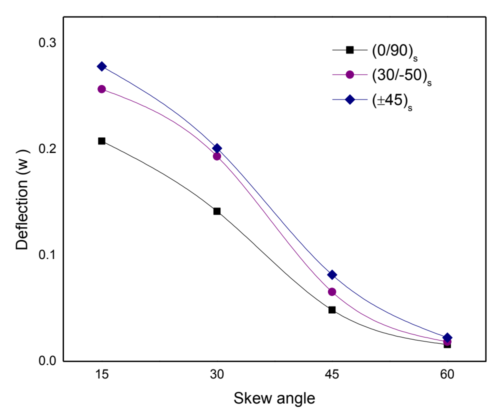

Fig. 42 Variation of *w* with skew angle and lamination scheme.

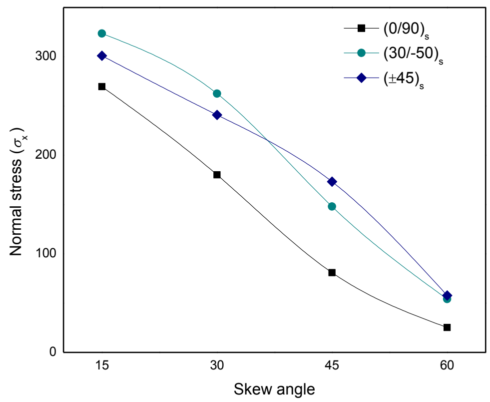

Fig. 43 Variation of σ_x with skew angle and lamination scheme.

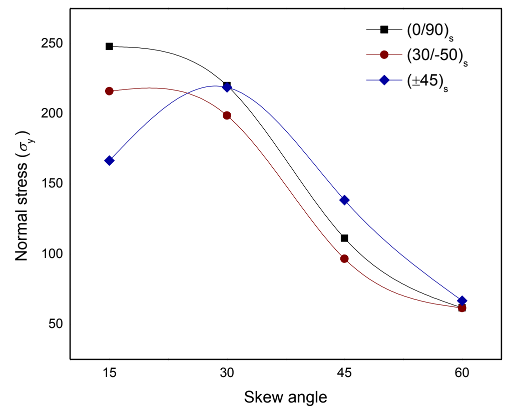

Fig. 44 Variation of σ_y with skew angle and lamination scheme.

different skew angles (α = 15˚, 30˚, 45˚ and 60˚) and four boundary conditions (SSSS, SSSC, SSCC and CCCC) at transverse pressure 0.5 MPa and plotted in Fig. 40. It can be seen that the deflections are showing higher and lower values at simply supported and clamped conditions, respectively. The other two boundary conditions (SSSC and SSCC) are showing an intermediate value.

Effect of four circular cutout diameters (d =2mm, 4 mm, 6mm and 8 mm) and four skew angles (α = 15˚, 30˚, 45˚ and 60˚) on the static behavior of clamped laminated composite cross ply (0˚/90˚) $_s$ skew plate under transverse pressure 0.5 MPa is plotted in Fig. 41. It can be seen that the deflections are decreasing as the both cutout size and skew angle increases.

Another new problem has been solved for clamped laminated skew plate subjected to four skew angles (α = 15˚, 30˚, 45˚ and 60˚) is analyzed for three lamination schemes [(0˚/90˚) $_s$, (30˚/-50˚) $_s$ and (±45˚) $_s$] under uniform transverse pressure of 0.5 MPa. The responses like deflection, normal and shear stresses of the skew plate have been presented by taking the circular cutout diameter of 4 mm. The deflections, normal stresses in x and y direction and shear stress in x-y plane for four different skew angles and three lamination schemes are plotted in Fig. 42-45. It can be concluded that the cross ply lamination (0˚/90˚) $_s$ is showing lower deflections whereas angle ply laminations (±45˚) $_s$ have higher for the same geometry, material, load and support.

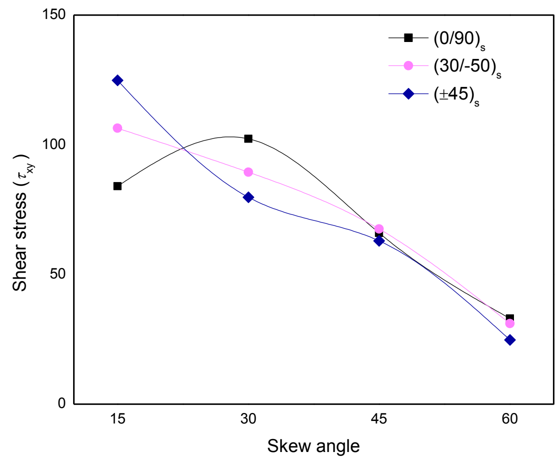

Fig. 45 Variation of τ_{xy} with transverse pressure and lamination scheme.

4.3.2.2 Free vibration analysis with and without cutout

In this section, effects of different parameters on the vibration responses of laminated composite skew plate with and without cutout are discussed in detail. The first four paragraphs discusses the skew plate without cutout (effect of boundary conditions, modular ratio, thickness ratio, lamination schemes and skew angles) whereas the last five regarding the skew plate with cutout (effect of boundary condition, lamination scheme, thickness ratio, cutout geometry and skew angle).

In this example, the nondimensional frequencies of a moderately thick (a/h = 10) angle ply $(\pm 45^\circ)_2$ skew plate at four different support conditions (SSSS, SSSC, SSCC and CCCC) and four skew angles (α = 15˚, 30˚, 45˚ and 60˚) are computed and plotted in Fig. 46. From this figure, it can be seen that the frequencies are showing higher and lower value at clamped and simply supported condition, respectively. The other two boundary conditions (SSSC and SSCC) are showing an intermediate value.

Another new problem has been solved to show the effect of modular ratio on the vibration behavior of laminated skew plate. In this analysis a simply supported angle ply $(\pm 45^\circ)_2$ moderately thick skew plate are analyzed for four different skew angles (α = 15˚, 30˚, 45˚ and 60˚) and the nondimensional frequencies with five modular ratio (E_1/E_2 = 3, 10, 20, 30 and 40)

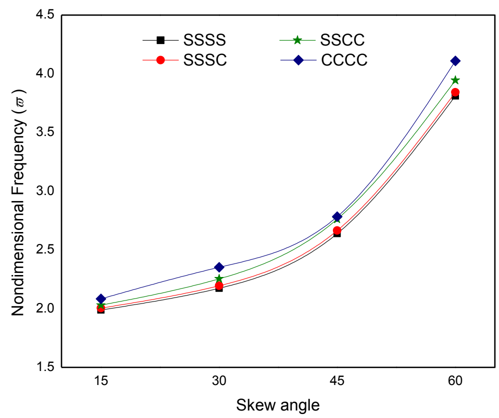

Fig. 46 Variation of $\bar{\omega}$ with different skew angle and boundary conditions.

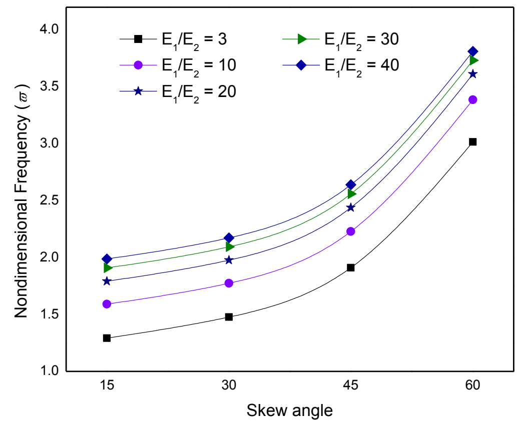

Fig. 47 Variation of $\bar{\omega}$ with different skew angle and modular ratio.

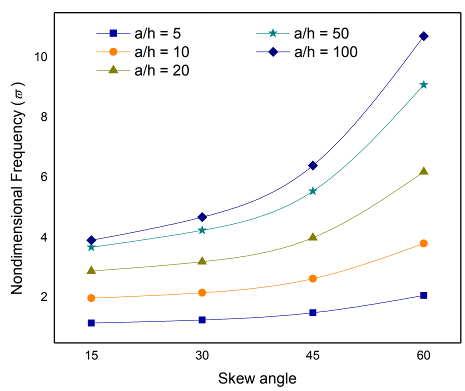

Fig. 48 Variation of $\bar{\omega}$ with different skew angle and thickness ratio.

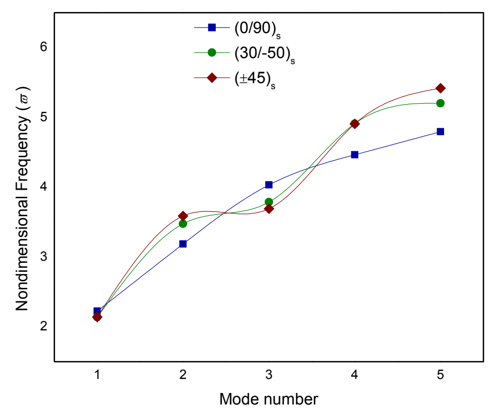

Fig. 49 Variation of $\bar{\omega}$ with different ply orientation and modes.

are plotted in Fig. 47 It can be seen from the figure that the frequencies are increasing as the modular ratio and skew angle increases.

It is well known that the composite properties are dependent on the thickness ratio. Hence, an example has been solved for simply supported angle ply $(\pm 45^{\circ})_2$ skew plate for four skew angles ($\alpha = 15^{\circ}$, 30°, 45° and 60°) and five thickness ratios (a/h = 5, 10, 20, 50, 100) and

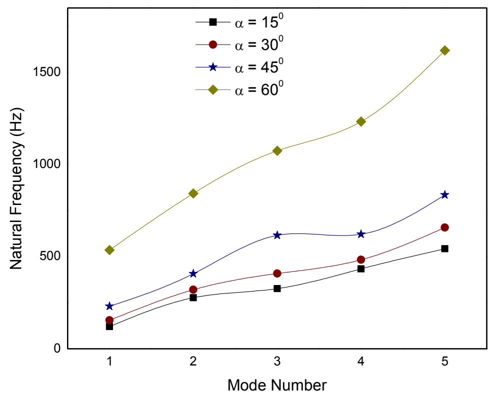

Fig. 50 Variation of $\bar{\omega}$ with different skew angle and modes.

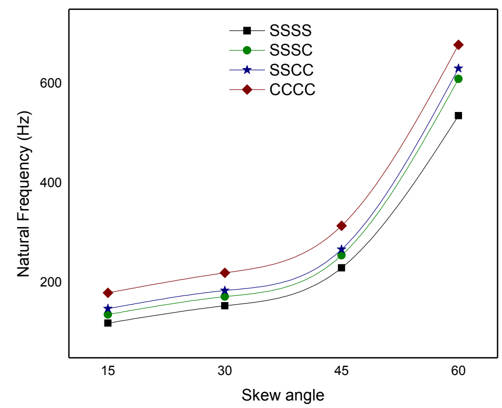

Fig. 51 Variation of $\bar{\omega}$ with different skew angle and boundary conditions.

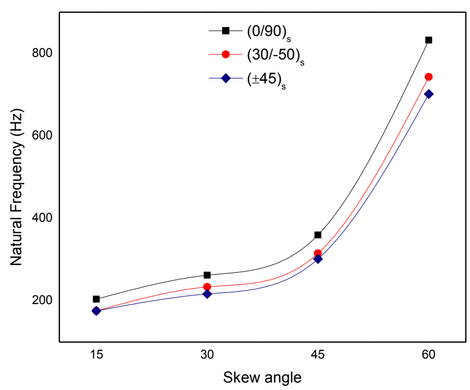

Fig. 52 Variation of $\bar{\omega}$ with different skew angle and ply orientation.

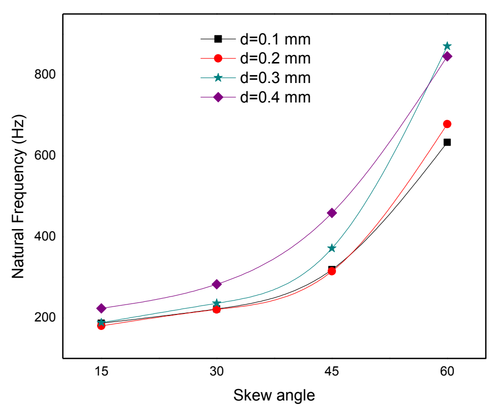

Fig. 53 Variation of $\bar{\omega}$ with different skew angle and ply cutout size.

the responses are plotted in Fig. 48. It can be seen that the frequencies increases for both thickness ratio and skew angle.

The nondimensional frequencies of five different mode of a moderately thick angle ply $(\pm45^\circ)_2$ clamped skew plate ($\alpha = 15^\circ$) at three different lamination schemes [$(0^\circ/90^\circ)_s$, $(30^\circ/-50^\circ)_s$ and $(\pm45^\circ)_s$] are computed and plotted in Fig. 49. The frequencies are higher for $(\pm45^\circ)_s$ lamination scheme and lower for $(0^\circ/90^\circ)_s$ laminates and the $(30^\circ/-50^\circ)_s$ have the intermediate value.

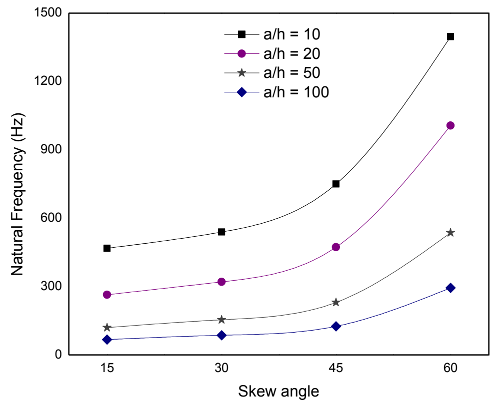

Fig. 54 Variation of $\overline{\omega}$ with different skew angle and thickness ratio.

A new problem has been solved to show the effect of cutout on the vibration behavior of laminated composite skew plate. The natural frequencies of different mode of simply supported anti-symmetric angle ply $(\pm 45)_2$ skew plates with circular cutout of diameter 0.2m and thickness ratio 50 is analyzed for four different skew angle (α = 15˚, 30˚, 45˚ and 60˚) and plotted in Fig. 50. It can be seen that the natural frequencies increases as the skew angle increases.

Effect of different support conditions (SSSS, SSSC, SSCC and CCCC) and four different skew angles (α = 15˚, 30˚, 45˚ and 60˚) on the laminated skew plate with circular cutout is analyzed in this example. The natural frequencies of angle ply $(\pm 45)_2$ skew plates are shown in Fig. 51. It can be seen from the figure that clamped conditions is showing the highest natural frequency and simply supported condition is the lowest and the other two boundary conditions (SSSC and SSCC) has the intermediate value.

A clamped skew plate of thickness ratio 50 and circular cutout diameter 0.2 m is analyzed for three lamination schemes [$(0^\circ/90^\circ)_s$, $(30^\circ/\text{-}50^\circ)_s$ and $(\pm 45^\circ)_s$] and four different skew angles (α = 15˚, 30˚, 45˚ and 60˚) and the natural frequencies are plotted in Fig. 52. The cross ply $(0^\circ/90^\circ)_s$ is showing the highest and angle ply $(\pm 45^\circ)_s$ is the lowest natural frequency. The $(30^\circ/\text{-}50^\circ)_s$ lamination scheme is showing an intermediate value and frequency is increasing as the skew angle increases.

Influence of four cutout sizes (d = 0.1 mm, 0.2 mm, 0.3 mm and 0.4 mm) and four skew angles (α = 15˚, 30˚, 45˚ and 60˚) on the vibration behavior of a clamped anti-symmetric angle ply $(\pm 45)_2$skew plate is plotted in Fig. 53. The frequency is increasing for both skew angle and cutout size.

A simply supported angle ply $(\pm 45)_2$skew plate with circular cutout (d = 0.2 m) is analyzed for four different thickness ratio (a/h = 10, 20, 50 and 100) and four skew angles (α = 15˚, 30˚, 45˚ and 60˚) and plotted in Fig. 54. It can be seen that the natural frequency decreases for moderately thick to thin skew plates.

5. CONCLUSIONS

Static and free vibration behavior of laminated composite plate/skew plate with and without cutouts is carried out using APDL code in ANSYS and validate with the available published results. From the present parametric analysis following conclusions are made:

- A convergence and validation study of the static and free vibration analysis of laminated composite plate/skew plate has been obtained. It is seen that the present model converging well with mesh refinement and the differences are within acceptable range.
- The normal stress values are higher for cross ply lamination scheme and lower for the angle ply scheme and the values are showing a reverse trend for shear stress for both (with and without cutout) the cases.
- The effect of cutout size shows that the deflections decreases with increase in size of cutout and the plate with rectangular cutout showing the maximum deflection as compared to other (square and circular) cutout.
- The frequency increases as number of layers increases and the values are showing higher for clamped condition and lower for the simply supported for both (with and without cutout) the conditions.
- The frequency increases with increase in the modular ratios and the thickness ratios.
- In the case of skew plate, the center deflection increases as the load parameter increases and the skew angle decreases.
- The effect of modular ratio and thickness ratio shows that the center deflection decreases as the both modular ratio and skew angle increases and it increases for thickness ratio increases and the skew angle decreases.
- The effect of cutout on skew plate shows that deflections are decreasing as the both cutout size and skew angle increases.
- The effect of modular and thickness ratio on the vibration behavior of skew plate is that the frequencies are increasing as the modular ratio and skew angle increases and it increase for both thickness ratio and skew angle.
- The natural frequency is increasing for both skew angle and cutout size and it decreases for moderately thick to thin skew plates.

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